

# Effect of Vacuum Properties on Electroweak Processes – A Theoretical Interpretation of Experiments

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Recently for discharges in fluids induced nuclear transmutations have been observed. It is our hypothesis that these reactions are due to a symmetry breaking of the electroweak vacuum by the experimental arrangement. The treatment of this hypothesis is based on the assumption that electroweak bosons, leptons and quarks possess a substructure of elementary fermionic constituents. The dynamical law of these fermionic constituents is given by a relativistically invariant nonlinear spinor field equation with local interaction, canonical quantization, selfregularization and probability interpretation. Phenomenological quantities of electroweak processes follow from the derivation of corresponding effective theories obtained by algebraic weak mapping theorems where the latter theories depend on the spinor field propagator, i.e. a vacuum expectation value. This propagator and its equation are studied for conserved and for broken discrete symmetries. For combined CP- and isospin symmetry breaking it is shown that the propagator corresponds to the experimental arrangements under consideration. The modifications of the effective electroweak theory due to this modified propagator are discussed. Based on these results a mechanism is sketched which offers a qualitative interpretation of the appearance of induced nuclear transmutations. A numerical estimate of electron capture is given.

*Key words:* Discharges in Fluids; Symmetry Breaking; Induced Nuclear Transitions.

## 1. Introduction

Recently low energy (electroweak) nuclear reaction rates have been reported which cannot be confirmed by calculations within the electroweak Standard Model [1–4]. It is our hypothesis that these reactions are the consequence of an experimental symmetry breaking of the vacuum.

Such an experimental manipulation of the vacuum has a mathematical counterpart: In the algebraic formulation of quantum field theory the behaviour of a system can be expressed by an infinite set of inequivalent representations, which are generated by an infinite set of inequivalent vacuum states [5].

This algebraic method has been successfully applied in quantum field theory of solids. There the various groundstates of matter constitute the set of inequivalent vacuum states, generated by different symmetry properties of the system [6–8], which frequently leads to a completely different physical behaviour of the system compared with that of the symmetry conserving groundstate.

To transfer this concept to nuclear and elementary particle physics a model has to be used which allows to perform nonperturbative calculations. Such a model was developed in the last decades. In a modern version of the fusion ideas of de Broglie and Heisenberg this model is designed to describe elementary particles with fermionic substructure, and its mathematical treatment is in accord with the basic ideas of the (non-perturbative) algebraic representation theory. In this approach the Standard Model is considered as an effective theory derived by weak mapping theorems which implies the chance to study processes beyond the Standard Model [9–11].

In the following treatment no use is made of the decomposition into left-handed and right-handed fermions for simplicity. Insofar the model leads to a simplified version of the mathematical structure of the Standard Model. This is justified as already in this version the crucial effects of changes of the vacuum by symmetry breaking can be demonstrated.

Concerning the nomenclature the fermionic constituents of the “elementary” particles are named “par-

tons” or “subfermions”. The former should not be identified with quarks, but are defined by the quantum numbers of the spinor fields which constitute the basic quantities of the model. Thus these fermionic constituents are only formal quantities which are applied to express the formal content of group theoretical and algebraic calculations in short terms. They are not to be identified with observable physical particles.

This paper is the fourth paper in a series of papers devoted to the explanation of the above problem. For the sake of brevity it is inevitable to refer to the previous papers without explicitly repeating their content. In particular for this paper the algebraic representation of the spinor field and the algebraic representation of effective theories are basic for its understanding. These topics have been treated in detail in [12], sections 2–4.

According to the formulas of [12], section 3 a corresponding effective theory contains the vacuum expectation value of the original spinor field which algebraically fixes the representation. In this paper the deductions are directly started with a discussion of this value characterized by the fermion propagator. *Owing to the selfregularization and the superspin-isospin structure this fermion propagator is not an ordinary Feynman propagator of the Dirac equation. To understand the effect of symmetry breaking one must therefore first study the superspin-isospin propagator for conserved symmetries.*

## 2. PCT- and CP-Invariant Fermion Propagators

In the formfactors of the effective theory, the influence of the vacuum is characterized by the fermion (parton) propagator  $F_{II'}$ . In the case that all symmetries of the Lagrangian are shared by the groundstate in preceding calculations the free superspin-isospin fermion propagator has been used. This propagator reads

$$F_{Z_1 Z_2}(x_1, x_2) = -i (2\pi)^{-4} \lambda_{i_1} \delta_{i_1 i_2} \gamma_{\kappa_1 \kappa_2}^5 \cdot \int d^4 p \left( \frac{\gamma^\mu p_\mu + m_{i_1}}{p^2 - m_{i_1}^2 + i\varepsilon} C \right)_{\alpha_1 \alpha_2} e^{-ip(x_1 - x_2)}, \quad (1)$$

where  $\kappa \equiv (\Lambda, A) = 1, 2, 3, 4$  is the superspin-isospin index arising from the combination  $\Lambda$ ,  $A = 1, 2$  (cf. [9], equation (6.7)).

For discrete transformations P, C and T the transformation properties of the general spinor fields have been discussed in [13]. These transformation formulas represent an extension of the corresponding formu-

las of the conventional theory by the inclusion of the superspin-isospin indices of the spinor fields.

In this formalism the action of a PCT-transformation on the spinor fields is defined by the antiunitary (or even more general) operator  $\mathcal{A} := \mathcal{PCT}$  and leads to ([13], equation (35))

$$\begin{aligned} \psi'(x) &:= \mathcal{A} \psi_{\kappa \alpha i}(x) \mathcal{A}^{-1} \\ &= (\gamma^5 \gamma^0)_{\kappa \kappa'} (\gamma^5 \gamma^0 C)_{\alpha \alpha'} \psi_{\kappa' \alpha' i}(x') \end{aligned} \quad (2)$$

with  $x' = -x$ . The formula (2) holds for free fields  $\chi$  too, as the transformation laws for free and for interacting fields are the same.

Furthermore by combination of the formulas (3) and (36) of [13], one obtains for the unitary PC-transformation the formula

$$\begin{aligned} \psi'(x) &= \mathcal{U} \psi_{\kappa \alpha i}(x) \mathcal{U}^{-1} \\ &= (\gamma^0 \gamma^5)_{\kappa \kappa'} \gamma_{\alpha \alpha'}^0 \psi_{\kappa' \alpha' i}(x') \end{aligned} \quad (3)$$

with  $\mathcal{U} := \mathcal{CP}$  and  $x' = (-\mathbf{r}, t)$ .

If these formulas are decomposed and retranslated into the conventional representation of the spinor fields, they coincide with those of [14–16].

The existence of such transformations implies relations between original and transformed matrix elements:

Let  $\mathcal{O}$  be any element of the field operator algebra and  $|a\rangle$  and  $|b\rangle$  elements of a corresponding state space, then for antiunitary transformations the relation

$$\langle a | \mathcal{O} | b \rangle = \langle a' | \mathcal{O}' | b' \rangle^* = \langle b' | (\mathcal{O}')^\dagger | a' \rangle \quad (4)$$

can be derived, cf. [14], equation (8.84), where the primed quantities are defined by  $|b'\rangle = \mathcal{A}|b\rangle$  and  $\mathcal{O}' = \mathcal{A} \mathcal{O} \mathcal{A}^{-1}$ . The transformations (2) and (4) are compatible, see [14], p. 225.

For unitary transformations one obtains for matrix elements

$$\langle a | \mathcal{O} | b \rangle = \langle a' | \mathcal{O}' | b' \rangle \quad (5)$$

with  $|b'\rangle = \mathcal{U}|b\rangle$  and  $\mathcal{O}' = \mathcal{U} \mathcal{O} \mathcal{U}^{-1}$ .

The covariant propagator (1) can be expressed as the special matrix element

$$F_{Z_1 Z_2}(x_1, x_2) := \langle 0 | \mathcal{O}_{Z_1 Z_2}(x_1, x_2) | 0 \rangle \quad (6)$$

with

$$\begin{aligned} \mathcal{O} &:= [\Theta(t_1 - t_2) \chi_{Z_1}(x_1) \chi_{Z_2}(x_2) \\ &\quad - \Theta(t_2 - t_1) \chi_{Z_2}(x_2) \chi_{Z_1}(x_1)], \end{aligned} \quad (7)$$

where  $|0\rangle$  is the physical groundstate of the system and  $\chi := \psi^f$  are the associated free spinor fields to the general fields  $\psi$ .

In Wigner's definition of the symmetry of a quantum system the matrix relations (4) and (5) can be used to show the invariance of the transition probability between states under such antiunitary or unitary transformations.

But Wigner's definition of the symmetry of quantum systems is not suitable for the characterization of the symmetry properties of the propagator. A better insight into these properties can be gained by the following definition which corresponds to the second active point of view, see [15], p. 45.

Consider the sets  $\{|b\rangle\}$  and  $\{\mathcal{A}|b\rangle \equiv |b'\rangle\}$  or  $\{\mathcal{U}|b\rangle \equiv |b'\rangle\}$  as different representation spaces of the field operator algebra and define by (6) and by

$$F_{Z_1 Z_2}(x_1, x_2)' := \langle 0' | \mathcal{O}_{Z_1 Z_2}(x_1, x_2) | 0' \rangle \quad (8)$$

the corresponding different representations of the operator  $\mathcal{O}_{Z_1 Z_2}(x_1, x_2)$ . Then the following theorem can be derived

**Proposition 1.** Under the change of the representation spaces for PCT- and CP-transformations the integral representation (1) of the free fermion propagator is forminvariant.

*Proof:* (i) According to the above transformation laws for states and operators the identity

$$\langle a' | \mathcal{O}' | b' \rangle^* = \langle a | \mathcal{A}^{-1} \mathcal{A} \mathcal{O} \mathcal{A}^{-1} \mathcal{A} | b \rangle^* \quad (9)$$

holds and is valid in particular for  $\mathcal{O}$  given by (7).

The operator  $\mathcal{A}$  acts exclusively in state space. Real  $c$ -number functions like  $\Theta$  are not changed by its application. Therefore (9) can be rewritten as

$$\begin{aligned} \langle 0' | \mathcal{O}' | 0' \rangle^* &= \Theta(t_1 - t_2) \\ &\cdot \langle 0' | \mathcal{A}_{\chi_{Z_1}}(x_1) \mathcal{A}^{-1} \mathcal{A}_{\chi_{Z_2}}(x_2) \mathcal{A}^{-1} | 0' \rangle^* \\ &- \Theta(t_2 - t_1) \langle 0' | \mathcal{A}_{\chi_{Z_2}}(x_2) \mathcal{A}^{-1} \mathcal{A}_{\chi_{Z_1}}(x_1) \mathcal{A}^{-1} | 0' \rangle^*. \end{aligned} \quad (10)$$

If in (10) intermediate states are introduced with  $\langle 0' | \chi'_Z(x') | b \rangle^* = \langle b | \chi'_Z(x')^+ | 0' \rangle$  one obtains from (4) and (10) the expression

$$\begin{aligned} \langle 0 | T[\chi_{Z_1}(x_1) \chi_{Z_2}(x_2)] | 0 \rangle &:= \\ \Theta(t_1 - t_2) \langle 0' | \chi'_{Z_2}(x_2)^+ \chi'_{Z_1}(x_1)^+ | 0' \rangle & \quad (11) \\ - \Theta(t_2 - t_1) \langle 0' | \chi'_{Z_1}(x_1)^+ \chi'_{Z_2}(x_2)^+ | 0' \rangle, \end{aligned}$$

where  $\chi'$  is the  $\mathcal{A}$ -transform of  $\chi$  and  $|0'\rangle = \mathcal{A}|0\rangle$ .

Because the right-hand side of (11) is no proper time-ordered product, it must be rearranged in order to allow a reasonable interpretation.

In superspinor-isospinor notation the general relation

$$\psi_{\kappa\alpha i}(x)^+ = -(\gamma^0 C)_{\alpha\alpha'} \gamma_{\kappa\kappa'}^5 \psi_{\kappa'\alpha' i}(x) \quad (12)$$

can be derived. This relation holds for any spinor field and thus also for  $\chi'(x)^+$ . If in addition in this case formula (2) is applied to  $\chi'(x)$ , combination of both formulas yields

$$\chi'_{\kappa\alpha i}(x)^+ = \gamma_{\kappa\kappa'}^0 \gamma_{\alpha\alpha'}^5 \chi_{\kappa'\alpha' i}(x'). \quad (13)$$

Substitution of (13) into (11) and replacement of  $t_i$  by  $-t'_i$  leads to the relation

$$\begin{aligned} F_{\alpha_1\alpha_2}^{\kappa_1\kappa_2}(x_1, x_2)_{i_1 i_2} &= -\gamma_{\kappa_1\kappa'_1}^0 \gamma_{\kappa_2\kappa'_2}^5 \gamma_{\alpha_1\alpha'_1}^5 \gamma_{\alpha_2\alpha'_2}^5 \\ &\cdot \langle 0' | T'[\chi_{\kappa'_1\alpha'_1 i_1}(x'_1) \chi_{\kappa'_2\alpha'_2 i_2}(x'_2)] | 0' \rangle. \end{aligned} \quad (14)$$

The right-hand side of (14) is the representation of the operator  $\mathcal{O}$  at the points  $x'_1, x'_2$  in the transformed state space with the transformed vacuum  $|0'\rangle$ . It is thus the propagator referred to these transformed states.

In accordance with (8) we therefore define

$$\begin{aligned} F_{\alpha_1\alpha_2}^{\kappa_1\kappa_2}(x'_1, x'_2)_{i'_1 i'_2} &:= \\ \langle 0' | T'[\chi_{\kappa_1\alpha_1 i_1}(x'_1) \chi_{\kappa_2\alpha_2 i_2}(x'_2)] | 0' \rangle, \end{aligned} \quad (15)$$

where time ordering  $T'$  is referred to the primed coordinates. Then (14) can be resolved for  $F'$  which gives

$$\begin{aligned} F_{\alpha_1\alpha_2}^{\kappa_1\kappa_2}(x'_1, x'_2)_{i'_1 i'_2} &:= \\ -\gamma_{\kappa_1\kappa'_1}^0 \gamma_{\kappa_2\kappa'_2}^5 \gamma_{\alpha_1\alpha'_1}^5 \gamma_{\alpha_2\alpha'_2}^5 F_{\alpha'_1\alpha'_2}^{\kappa'_1\kappa'_2}(x_1, x_2)_{i_1 i_2}. \end{aligned} \quad (16)$$

If on the right-hand side of (16) the integral representation (1) of the propagator is substituted, it can be verified that  $F'$  has the same integral representation at the corresponding points  $x'_1, x'_2$ , i. e.  $F$  is forminvariant under PCT-transformation.

(ii) Concerning the CP-transformation, it is unitary in contrast to the antiunitary PCT-transformation. In this case in addition to the transformation of the field operators no special transformation for the matrix elements is required. Then with (5) one gets the propagator relation

$$\begin{aligned} \langle 0 | T[\chi_{Z_1}(x_1) \chi_{Z_2}(x_2)] | 0 \rangle & \\ = \langle 0' | T[\chi'_{Z_1}(x_1) \chi'_{Z_2}(x_2)] | 0' \rangle & \equiv F' \end{aligned} \quad (17)$$

as  $\mathcal{U}$  commutes with time ordering.

Owing to (3), (17) can equivalently be written:

$$F_{\alpha_1\alpha_2}^{\kappa_1\kappa_2}(x_1, x_2)_{i_1i_2} = (\gamma^0\gamma^5)_{\kappa_1\kappa'_1}(\gamma^0\gamma^5)_{\kappa_2\kappa'_2}\gamma_{\alpha_1\alpha'_1}^0 \cdot \gamma_{\alpha_2\alpha'_2}^0 F_{\alpha'_1\alpha'_2}^{\kappa'_1\kappa'_2}(x'_1, x'_2)'_{i_1i_2}. \quad (18)$$

This relation can be resolved for  $F'$ . If in the resulting equation the integral representation (1) for  $F$  is substituted, the algebra can be directly evaluated. In the resulting integral the transformation  $p' = (-\mathbf{p}, p_0)$  can be performed which eventually gives  $F(x'_1, x'_2)$ . Therefore one obtains from this relation

$$F_{\alpha_1\alpha_2}^{\kappa_1\kappa_2}(x'_1, x'_2)'_{i_1i_2} = F_{\alpha_1\alpha_2}^{\kappa_1\kappa_2}(x'_1, x'_2)_{i_1i_2}, \quad (19)$$

which means forminvariance under CP-transformation.  $\diamond$

For the further proceeding we employ the inhomogeneous Dirac equation which is satisfied by the free fermion propagator (1). In the notation of [12], section 1, or [9], equation (3.109), it reads

$$[D^\mu\partial_\mu(x_1) - m]_{ZZ_1} F_{Z_1Z_2}(x_1, x_2) = D_{ZZ_1}^0 A_{Z_1Z_2}\delta(x_1 - x_2). \quad (20)$$

For this equation the following theorem holds.

**Proposition 2.** The propagator equation (20) is forminvariant under PCT- and CP-transformations.

*Proof:* With explicit indexing (20) reads

$$[i\gamma_{\alpha\alpha_1}^\mu\partial_\mu(x_1) - m_i\delta_{\alpha\alpha_1}]\delta_{\kappa\kappa_1}\delta_{ii_1}F_{\alpha_1\alpha_2}^{\kappa_1\kappa_2}(x_1, x_2)_{i_1i_2} = -iC_{\alpha\alpha_2}\gamma_{\kappa\kappa_2}^5\lambda_i\delta_{ii_2}\delta(x_1 - x_2). \quad (21)$$

(i) For a PCT-transformation combination of (14) and (15) and substitution of the resulting expression in (21) yields

$$[i\gamma_{\alpha\alpha_1}^\mu\partial_\mu(x_1) - m_i\delta_{\alpha\alpha_1}]\delta_{\kappa\kappa_1}\delta_{ii_1} \cdot \gamma_{\kappa_1\kappa'_1}^0\gamma_{\kappa_2\kappa'_2}^0\gamma_{\alpha_1\alpha'_1}^5\gamma_{\alpha_2\alpha'_2}^5F_{\alpha'_1\alpha'_2}^{\kappa'_1\kappa'_2}(x'_1, x'_2)'_{i_1i_2} \quad (22) \\ = iC_{\alpha\alpha_2}\gamma_{\kappa\kappa_2}^5\lambda_i\delta_{ii_2}\delta(x'_1 - x'_2).$$

Elimination of the four transformation matrices  $\gamma^0$  etc. by multiplication of the whole equation with their dual matrices and with the transformation of  $x = -x'$  in the partial derivative, this equation yields

$$[i\gamma_{\beta_1\alpha'_1}^\mu\partial_\mu(x'_1) - m_i\delta_{\beta_1\alpha'_1}]\delta_{ii_1}\delta_{\lambda_1\kappa'_1}F_{\alpha'_1\beta_2}^{\kappa'_1\lambda_2}(x'_1, x'_2)'_{i_1i_2} \\ = -iC_{\beta_1\beta_2}\gamma_{\lambda_1\lambda_2}^5\lambda_i\delta_{ii_2}\delta(x'_1 - x'_2), \quad (23)$$

i. e., forminvariance of (20) under PCT-transformation.

(ii) Concerning the CP-transformation, we substitute relation (18) into (21). This gives

$$[i\gamma_{\alpha\alpha_1}^\mu\partial_\mu(x_1) - m_i\delta_{\alpha\alpha_1}]\delta_{\kappa\kappa_1}\delta_{ii_1}(\gamma^0\gamma^5)_{\kappa_1\kappa'_1} \cdot (\gamma^0\gamma^5)_{\kappa_2\kappa'_2}\gamma_{\alpha_1\alpha'_1}^0\gamma_{\alpha_2\alpha'_2}^0F_{\alpha'_1\alpha'_2}^{\kappa'_1\kappa'_2}(x'_1, x'_2)'_{i_1i_2} \quad (24) \\ = -iC_{\alpha\alpha_2}\gamma_{\kappa\kappa_2}^5\lambda_i\delta_{ii_2}\delta(x_1 - x_2)$$

with  $x' = (-\mathbf{r}, t)$ . Eliminating the four transformation matrices  $(\gamma^0\gamma^5)$  etc. by multiplication of the whole equation with their dual matrices and with  $\mathbf{r} = -\mathbf{r}'$  in the partial derivative and the  $\delta$ -distribution, one eventually obtains

$$[i\gamma_{\alpha\alpha_1}^\mu\partial_\mu(x'_1) - m_i\delta_{\alpha\alpha_1}]\delta_{\kappa\kappa_1}\delta_{ii_1}F_{\alpha_1\alpha_2}^{\kappa_1\kappa_2}(x'_1, x'_2)'_{i_1i_2} \\ = -iC_{\alpha\alpha_2}\gamma_{\kappa\kappa_2}^5\lambda_i\delta_{ii_2}\delta(x'_1 - x'_2), \quad (25)$$

i. e. the propagator equation is forminvariant under CP-transformation.  $\diamond$

Next we consider the propagator and its equation under the influence of an electromagnetic field. The propagator is referred to superspin-isospin states of parton fields which means that the coupling of partons to an electromagnetic field has to be derived. The effective  $SU(2) \otimes U(1)$ -gauge theory of composite electroweak vector bosons in interaction with partons was extensively studied by weak mapping calculations in [17, 18]. This effective theory is identical with the corresponding phenomenological gauge theory of the interaction of electroweak gauge bosons with fermions. But to get the correct coupling terms of fermions to neutral and charged vector bosons the gauge symmetry has to be broken.

For the sake of brevity we skip the corresponding calculations which among other things lead to the covariant derivative for the coupling of partons to electromagnetic fields:

$$D_\mu := \partial_\mu - ieQ_{\kappa\kappa'}A_\mu(x), \quad (26)$$

where the charge operator  $Q$  of the parton fields  $\psi$  is given in superspinor-isospinor representation by (see [9], equation (6.103))

$$Q_{\kappa\kappa'} := \frac{1}{3} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \equiv \frac{1}{6}(\gamma^0 - 3\gamma^5\gamma^3)_{\kappa\kappa'}. \quad (27)$$

Therefore due to (26), (27) in this case the propagator

equation reads

$$\begin{aligned} & \{[i \gamma_{\alpha\alpha_1}^\mu \partial_\mu(x_1) - m_i \delta_{\alpha\alpha_1}] \delta_{\kappa\kappa_1} \\ & + e \gamma_{\alpha\alpha_1}^\mu Q_{\kappa\kappa_1} A_\mu(x_1)\} \delta_{ii_1} F_{\alpha_1\alpha_2}^{\kappa_1\kappa_2}(x_1, x_2)_{i_1 i_2} \quad (28) \\ & = -i C_{\alpha\alpha_2} \gamma_{\kappa\kappa_2}^5 \lambda_i \delta_{ii_2} \delta(x_1 - x_2). \end{aligned}$$

Then for PCT- and CP-transformations of (28) the following result is obtained.

**Proposition 3.** The propagator equation (28) is forminvariant under PCT- and CP-transformations provided that the relations

$$\begin{aligned} \mathcal{A} A_\mu(x) \mathcal{A}^{-1} &= -A_\mu(-x), \\ \mathcal{U} A_\mu(x) \mathcal{U}^{-1} &= -\eta^{\mu\mu'} A_{\mu'}(-\mathbf{r}, t) \end{aligned} \quad (29)$$

are satisfied.

*Proof:* The proof runs along the lines of Proposition 2. Hence in this context we only have to study the coupling term of (28).

(i) If a PCT-transformation, (14) or (15), is substituted into (28), the minus sign of (14) is removed from (28) by multiplication of the whole equation with  $(-1)$ . Thus one obtains for the transformed coupling term

$$\begin{aligned} & e \gamma_{\alpha\alpha_1}^\mu Q_{\kappa\kappa_1} A_\mu(x_1) \gamma_{\kappa_1\kappa'_1}^0 \gamma_{\kappa_2\kappa'_2}^0 \gamma_{\alpha_1\alpha'_1}^5 \gamma_{\alpha_2\alpha'_2}^5 \delta_{ii_1} \\ & \cdot F_{\alpha'_1\alpha'_2}^{\kappa'_1\kappa'_2}(x'_1, x'_2)_{i'_1 i'_2} = -e \gamma_{\kappa\kappa_1}^0 \gamma_{\kappa_2\kappa'_2}^0 \gamma_{\alpha\alpha_1}^5 \gamma_{\alpha_2\alpha'_2}^5 \\ & \cdot \gamma_{\alpha_1\alpha'_1}^\mu Q_{\kappa_1\kappa'_1} A_\mu(x_1) \delta_{ii_1} F_{\alpha'_1\alpha'_2}^{\kappa'_1\kappa'_2}(x'_1, x'_2)_{i'_1 i'_2}, \end{aligned} \quad (30)$$

as in the first line the minus sign of (14) drops out and as  $[Q, \gamma^0]_- = 0$ .

Furthermore from (29) it follows with  $A'_\mu(x) \equiv \mathcal{A} A_\mu(x) \mathcal{A}^{-1}$  that  $-A'_\mu(-x) = A_\mu(x)$  or equivalently  $A_\mu(x) = -A'_\mu(x')$ . If the latter relation is substituted into the second line of (30) and if in analogy to Proposition 2 the transformation matrices are eliminated from the propagator equation, the transformed equation

$$\begin{aligned} & \{[i \gamma_{\beta\alpha'_1}^\mu \partial_\mu(x'_1) - m_i \delta_{\beta\alpha'_1}] \delta_{\lambda\lambda'_1} \\ & + e \gamma_{\beta\alpha'_1}^\mu Q_{\lambda_1\lambda'_1} A'_\mu(x'_1)\} \delta_{ii_1} F_{\alpha'_1\beta_2}^{\kappa'_1\lambda_2}(x'_1, x'_2)_{i'_1 i_2} \quad (31) \\ & = -i C_{\beta_1\beta_2} \gamma_{\lambda_1\lambda_2}^5 \lambda_i \delta_{ii_2} \delta(x'_1 - x'_2) \end{aligned}$$

results, i. e., (28) is forminvariant under PCT-transformations.

(ii) In a similar way the forminvariance under PC-transformations of (28) can be verified.  $\diamond$

*Addendum:* The transformation relations (29) coincide with those of the phenomenological theory, see [15], equations (6.276), (6.149), (6.164a).

### 3. Propagator for Symmetry Breaking Experiments

The corresponding experiments of Urutskoev et al. are rather intricate. Discharges between metallic foils in vessels filled with various fluids lead to the evidence of numerous elements being not present in the system before the explosion and depending on the special foils and fluids. In spite of a lot of interesting semiempirical studies, the physical mechanism underlying these low energy nuclear reactions is unknown. Only the action of strong forces is definitely excluded.

The authors assume that their experiments might be connected with a violation of the conventional electroweak reaction schemes, possibly triggered by light magnetic monopoles as discussed in [4]. But so far no theoretical formalism has been developed to describe such reactions.

In accordance with the statements in the introduction we assume that these experiments should be interpreted as manifestations of a symmetry breaking of the vacuum, leading to a new inequivalent vacuum state, a view which was already put forward in three preceding papers [12, 19, 20].

As a suitable candidate for this symmetry breaking an experimental violation of the commonly assumed CP-invariance has been considered. Hence in the following we try to show how this symmetry breaking comes about and how the theoretical counterpart of the experimental arrangement has to be formulated.

First we give a further statement about the CP-invariant superspin-isospin propagator.

**Proposition 4.** For free spinor fields  $\chi_Z$  the associated superspin-isospin propagator  $F_{Z_1 Z_2}(x_1 - x_2)$  can be decomposed into the sum of the conventional fermion and the conventional antifermion propagators at any time  $\tau := t_1 - t_2$ .

*Proof:* The index  $Z$  is defined by  $Z := (A; A, \alpha, i)$ . For the proof the indices  $A$  and  $i$  are spectator indices and will be suppressed for the sake of brevity:

$$\begin{aligned} & F_{A_1 A_2}^{\alpha_1 \alpha_2}(x_1 - x_2) := \\ & \Theta(t_1 - t_2) \langle 0 | \chi_{A_1 \alpha_1}(x_1) \chi_{A_2 \alpha_2}(x_2) | 0 \rangle \quad (32) \\ & + \Theta(t_2 - t_1) \langle 0 | \chi_{A_2 \alpha_2}(x_2) \chi_{A_1 \alpha_1}(x_1) | 0 \rangle. \end{aligned}$$

(i) Without loss of generality we assume  $t_1 - t_2 > 0$ . Then (18) reads

$$\begin{aligned} F_{A_1 A_2}^{\alpha_1 \alpha_2}(x_1 - x_2) &= \langle 0 | \chi_{A_1 \alpha_1}(x_1) \chi_{A_2 \alpha_2}(x_2) | 0 \rangle \\ &= \delta_{A_1 1} \delta_{A_2 2} \langle 0 | \chi_{1 \alpha_1}(x_1) \chi_{2 \alpha_2}(x_2) | 0 \rangle \\ &\quad + \delta_{A_1 2} \delta_{A_2 1} \langle 0 | \chi_{2 \alpha_1}(x_1) \chi_{1 \alpha_2}(x_2) | 0 \rangle \\ &= \delta_{A_1 1} \delta_{A_2 2} \langle 0 | \chi_{\alpha_1}(x_1) \chi_{\alpha_2}^c(x_2) | 0 \rangle \\ &\quad + \delta_{A_1 2} \delta_{A_2 1} \langle 0 | \chi_{\alpha_1}^c(x_1) \chi_{\alpha_2}(x_2) | 0 \rangle. \end{aligned} \quad (33)$$

For free fields the vacuum is invariant under discrete transformations  $\mathcal{C}$ ,  $\mathcal{P}$ ,  $\mathcal{C}$ ,  $\mathcal{T}$ . This means  $\mathcal{C}|0\rangle \equiv |0\rangle$  and

$$\begin{aligned} &\langle 0 | \chi_{\alpha_1}^c(x_1) \chi_{\alpha_2}(x_2) | 0 \rangle \\ &= \langle 0 | \mathcal{C}^{-1} \mathcal{C} \chi_{\alpha_1}^c(x_1) \mathcal{C}^{-1} \mathcal{C} \chi_{\alpha_2}(x_2) \mathcal{C}^{-1} \mathcal{C} | 0 \rangle \\ &= \langle 0 | \chi_{\alpha_1}(x_1) \chi_{\alpha_2}^c(x_2) | 0 \rangle. \end{aligned} \quad (34)$$

Hence both parts in (33) are identical.

Furthermore the two parts have to be shown to be particle and antiparticle propagators.

(ii) According to [21], section 15c it is  $\chi = \chi^+ + \chi^-$ , and one gets

$$\begin{aligned} &\langle 0 | \chi_{\alpha_1}(x_1) \chi_{\alpha_2}^c(x_2) | 0 \rangle \\ &= C_{\alpha_2 \alpha_2'} \langle 0 | \chi_{\alpha_1}(x_1) \bar{\chi}_{\alpha_2'}(x_2) | 0 \rangle \\ &= -i C_{\alpha_2 \alpha_2'} S_{\alpha_1 \alpha_2'}^+(x_1 - x_2). \end{aligned} \quad (35)$$

In the same way it follows that

$$\begin{aligned} &\langle 0 | \chi_{\alpha_1}^c(x_1) \chi_{\alpha_2}(x_2) | 0 \rangle \\ &= -i C_{\alpha_1 \alpha_1'} S_{\alpha_2 \alpha_1'}^-(x_2 - x_1), \end{aligned} \quad (36)$$

where  $S^+$  is the conventional particle propagator, while  $S^-$  is the conventional antiparticle propagator.  $\diamond$

Based on this theorem one can analyze the modifications which are necessary to adapt the theoretical description of the vacuum to the experimental arrangements. Two properties are characteristic for these experiments:

(i) the reactions are confined to the interior of a closed vessel;

(ii) the reactions by discharges proceed within a fluid medium.

We concentrate on the theoretical description of (ii) because in contrast to (i) an essential change of the properties of the vacuum has to be expected. This is due to the fact that for discharges in fluid media the motion of the charge carriers is damped, i. e. accompanied by energy losses. One of the standard fluids used

in these experiments is water. To be definite we refer our arguments to water.

The stopping power of matter for fast particles has been extensively discussed in [22], section 23. For electrons in water the following ranges have been calculated:

Primary energy	0.1	1	10	100	1000	$mc^2$
H <sub>2</sub> O	$0.47 \cdot 10^{-2}$	0.19	2.6	19	78	cm

For low energies the magnitudes of these ranges fit into the dimensions of the vessels in the above experiments.

The fact that positrons can be annihilated somewhere in their paths diminishes the average ranges of positrons in comparison with those of electrons. Corresponding formulas describing these differences are given in [22], section 23, equations (21), (22). Apart from the importance of numerical values, the principal effect of these differences consists in the signal of a symmetry breaking. Charged particles and their antiparticles behave differently in a medium which leads to  $\mathcal{C}$ - or  $\mathcal{CP}$ -symmetry breaking, respectively.

Discharges are triggered by electrons which on their way in the fluid ionize molecules, generate secondary electrons, etc. But to avoid theoretical difficulties in the description of the rather complicated processes of a discharge, we simplify the theoretical treatment by considering only an average damping effect of electrons.

In such discharges no positrons occur, and their real presence is not necessary, because the inclusion of positrons is a theoretical concept to study the behaviour of the system under charge conjugation. The absence of positrons in experiments is therefore no argument against positrons in the theoretical treatment.

In our model electrons and positrons are assumed to have a fermionic substructure. Therefore the question is: do the above considerations apply to their fermionic constituents as well?

The group theoretical representation of lepton states with respect to superspin-isospin combinations is exact. In particular for the superspin part one obtains (see [12], section 4)

$$e^+ \rightarrow \delta_{1A_1} \delta_{1A_2} \delta_{1A_3} \text{ and } e^- \rightarrow \delta_{2A_1} \delta_{2A_2} \delta_{2A_3}, \quad (37)$$

where  $A$  is the superspinor index. With respect to this index the fermion number  $f$  is defined. With these fermion numbers one gets for  $e^+$  the configuration  $(1/3, 1/3, 1/3)$  while  $e^-$  leads to the set

$(-1/3, -1/3, -1/3)$ . Fermion numbers are used to discriminate particles from antiparticles by changing  $f$  into  $-f$  by convention.

Thus from the above sets of fermion numbers it follows: electrons consist only of particles, positrons only of antiparticles. In the complete representation of the wave functions this property is not changed. Therefore the partons of the electrons and the antipartons of the positrons share their behaviour with that of electrons or positrons, respectively, and we can base our arguments on the spinor field propagator (1) instead of the phenomenological electron-positron Feynman propagator.

In the next step we consider the influence of damping on the motion of partons and antipartons. Their motion is described by the propagator, and if damping is effective their motion ceases in a finite time interval. This fact can be expressed by a damping factor in the integral representation (1) of the propagator. According to Feynman this integral can be evaluated by giving the mass an infinitesimal negative imaginary part, i. e.,  $m \rightarrow m - i\delta$ ,  $\delta > 0$ . If  $\delta$  is allowed to have a finite value this leads for  $t_1 - t_2 > 0$  to a damping factor  $\exp[-\delta'(t_1 - t_2)]$  with  $\delta' := m\delta$ , while for  $t_1 - t_2 < 0$  one obtains the damping factor  $\exp[\delta'(t_1 - t_2)]$  in the space-time representation of the propagator.

The damping factor introduced in this way is independent of superspin-isospin states, and it does not allow a different behaviour of partons and antipartons. But according to Proposition 4 the propagator (1) can be decomposed into a pure particle and a pure antiparticle propagator. Thus these propagators can be treated separately with different damping factors which leads to the experimentally observed different behaviour of particles and antiparticles in the fluid. Formally this can be described by giving in (20) the mass a superspin-isospin-dependent damping factor which respects the above decomposition.

According to [22] the damping factors of particles and antiparticles differ only weakly compared with their average absolute values. We therefore use the following formulation of this fact:

$$\delta_{ii'} \delta_{\kappa\kappa'} \delta_{\alpha\alpha'} m \rightarrow (m^* \delta_{\kappa\kappa'} - i \delta \gamma_{\kappa\kappa'}^0) \delta_{\alpha\alpha'} \delta_{ii'} \quad (38)$$

with  $m^* := m - i\delta^*$ , where  $\delta^*$  is the average damping factor, while  $\delta$  means the small difference between the damping factors of particles and antiparticles.

**Proposition 5.** In the propagator equation (21) the mass term (38) violates the CP-forminvariance of the propagator and of the equation.

*Proof:* According to Propositions 1 and 2 the propagator equation (21) for mass  $m$  and the propagator itself are forminvariant under CP-transformation. Therefore if in (21) the mass  $m$  is replaced by (38) a violation of CP-invariance can only be caused by this modified mass term.

The CP-transformation of the propagator equation (21) with mass term (38) is carried out by substituting (18) into (21) and subsequent elimination of the transformation matrices by multiplication of the whole equation with duals of these matrices. This gives for the modified mass term the following expression:

$$\begin{aligned} & (\gamma^0 \gamma^5)_{\lambda\kappa}^+ (\gamma^0 \gamma^5)_{\lambda_2 \kappa_2}^+ \gamma_{\beta\alpha}^0 \gamma_{\beta_2 \alpha_2}^0 (m^* \delta_{\kappa\kappa_1} - i \delta \gamma_{\kappa\kappa_1}^0) \\ & \cdot \delta_{\alpha\alpha_1} \delta_{ii_1} (\gamma^0 \gamma^5)_{\kappa_1 \kappa'_1} (\gamma^0 \gamma^5)_{\kappa_2 \kappa'_2} \gamma_{\alpha_1 \alpha'_1}^0 \gamma_{\alpha_2 \alpha'_2}^0 \\ & \cdot F_{\alpha'_1 \alpha'_2}^{\kappa'_1 \kappa'_2} (x'_1, x'_2)'_{i_1 i_2} \\ & \equiv (m^* \delta_{\lambda\kappa'_1} + i \delta \gamma_{\lambda\kappa'_1}^0) \delta_{\beta\alpha'_1} \delta_{ii_1} F_{\alpha'_1 \beta_2}^{\kappa'_1 \lambda_2} (x'_1, x'_2)'_{ii_2}. \end{aligned} \quad (39)$$

If in this way the whole equation is transformed one obtains

$$\begin{aligned} & [i \gamma_{\beta\alpha'_1}^\mu \partial_\mu (x'_1) \delta_{\lambda\kappa'_1} - (m^* \delta_{\lambda\kappa'_1} + i \delta \gamma_{\lambda\kappa'_1}^0) \delta_{\beta\alpha'_1}] \\ & \cdot \delta_{ii_1} F_{\alpha'_1 \beta_2}^{\kappa'_1 \lambda_2} (x'_1, x'_2)'_{i_1 i_2} \\ & = -i C_{\beta\beta_2} \gamma_{\lambda\lambda_2}^5 \lambda_i \delta_{ii_2} \delta(x'_1 - x'_2). \end{aligned} \quad (40)$$

By an appropriate change of indexing one can reestablish the original denotation of (21). Then a comparison between (21) with mass (38) and (40) shows: The damping term has changed its sign under CP-transformation, i. e. CP-invariance is violated. This holds for the solution of (21), i. e. the propagator too.  $\diamond$

Apart from discrete symmetry operations the propagator equation (21) admits the application of the continuous SU(2)-isospin group and the abelian U(1)-fermion number group. We are particularly interested in the isospin group.

**Proposition 6.** The propagator equation (21) is forminvariant under global isospin transformations.

*Proof:* We apply the decomposition of the superspin-isospin index  $\kappa$  into the pair of indices  $(A, A)$  where the index  $A$  enumerates isospin states.

To verify the statement, the charge-conjugated spinor fields  $\chi^c$  have to be replaced by  $G$ -parity spinors  $\chi^G$ . The latter are defined by

$$\chi_A^G := c_{AA'}^{-1} \chi_{A'}^c \quad (41)$$

with  $c := -i\sigma_2$ . In contrast to  $\chi^c$  the  $G$ -parity spinor (41) transforms cogredient to  $\chi$  for isospin transformations. This guarantees a homogenous isospin transformation law for superspinor-isospinor fields.

Denoting the pair  $(\chi, \chi^G)$  by  $\tilde{\chi}$ , one can express the relation between the superspinor-isospinor fields  $\chi_\kappa$  and  $\tilde{\chi}_\kappa$  by

$$\chi_\kappa = G_{\kappa\kappa'} \tilde{\chi}_{\kappa'}; \quad G := \begin{pmatrix} 1 & 0 \\ 0 & c \end{pmatrix}. \quad (42)$$

If in the definition of the propagator (6) and (7) relation (42) is substituted this leads to the transformation

$$\begin{aligned} F_{\alpha_1\alpha_2}^{\kappa_1\kappa_2}(x_1, x_2)_{i_1i_2} \\ = G_{\kappa_1\kappa'_1} G_{\kappa_2\kappa'_2} \tilde{F}_{\alpha_1\alpha_2}^{\kappa'_1\kappa'_2}(x_1, x_2)_{i_1i_2}, \end{aligned} \quad (43)$$

where  $\tilde{F}$  is referred to superspinor-isospinor fields  $\tilde{\chi}$ .

Substitution of relation (43) into (21) and elimination of the transformation matrices  $G$  yields the equation

$$\begin{aligned} [i\gamma_{\alpha\alpha_1}^\mu \partial_\mu(x_1) - m_i \delta_{\alpha\alpha_1}] \\ \cdot \delta_{\lambda\kappa'_1} \delta_{ii_1} \tilde{F}_{\alpha_1\alpha_2}^{\kappa'_1\lambda_2}(x_1, x_2)_{i_1i_2} \\ = -\gamma_{\lambda\lambda_2}^2 C_{\alpha\alpha_2} \lambda_i \delta_{ii_2} \delta(x_1 - x_2), \end{aligned} \quad (44)$$

i. e., the propagator equation in the field representation  $\tilde{\chi}$ . Owing to the homogenous isospin transformation of  $\tilde{\chi}$ , the isospin transformation properties of (44) can be analyzed. If one returns to the double indexing  $(\Lambda, A)$  and if the index value  $\Lambda = 2$  is now related to  $\chi^G$ , (44) can be written in the form

$$\begin{aligned} [i\gamma_{\alpha\alpha_1}^\mu \partial_\mu(x_1) - m_i \delta_{\alpha\alpha_1}] \delta_{\Lambda\Lambda_1} \delta_{AA_1} \delta_{ii_1} \\ \cdot \tilde{F}_{\alpha_1\alpha_2}^{\Lambda_1 A_1 \Lambda_2 A_2}(x_1, x_2)_{i_1i_2} \\ = -i\sigma_{\Lambda\Lambda_2}^2 \sigma_{AA_2}^2 C_{\alpha\alpha_2} \lambda_i \delta_{ii_2} \delta(x_1 - x_2). \end{aligned} \quad (45)$$

It is sufficient to study the invariance properties of (45) under infinitesimal transformations which are given by

$$dU := 1 - i \frac{\varepsilon^a}{n} \sigma_a, \quad dU^{-1} := 1 + i \frac{\varepsilon^a}{n} \sigma_a, \quad n \rightarrow \infty. \quad (46)$$

Then one obtains the following relation between  $\tilde{F}$  and its isospin transform  $\tilde{F}'$ :

$$\begin{aligned} \tilde{F}_{\alpha_1\alpha_2}^{\Lambda_1 A_1 \Lambda_2 A_2}(x_1, x_2)_{i_1i_2} \\ = dU_{A_1 A'_1} dU_{A_2 A'_2} \tilde{F}_{\alpha_1\alpha_2}^{\Lambda_1 A'_1 \Lambda_2 A'_2}(x_1, x_2)_{i'_1 i'_2}. \end{aligned} \quad (47)$$

Substitution of (47) into (45) and elimination of the transformation matrices yields with

$$dU_{BA}^{-1} dU_{B_2 A_2}^{-1} \sigma_{AA_2}^2 = \sigma_{BB_2}^2 + O\left(\left(\frac{\varepsilon}{n}\right)^2\right) \quad (48)$$

the equation

$$\begin{aligned} [i\gamma_{\alpha\alpha_1}^\mu \partial_\mu(x_1) - m_i \delta_{\alpha\alpha_1}] \delta_{\Lambda\Lambda_1} \delta_{BB_1} \delta_{ii_1} \\ \cdot \tilde{F}_{\alpha_1\alpha_2}^{\Lambda_1 B_1 \Lambda_2 B_2}(x_1, x_2)_{i'_1 i'_2} \\ = -i\sigma_{\Lambda\Lambda_2}^2 \sigma_{BB_2}^2 C_{\alpha\alpha_2} \lambda_i \delta_{ii_2} \delta(x_1 - x_2). \end{aligned} \quad (49)$$

In the limit  $n \rightarrow \infty$  the last term in (48) vanishes. This means for infinitesimal isospin transformations (49) is identical with (45). On account of the group properties of infinitesimal transformations this result holds for finite transformations too, i. e., (45) is forminvariant under these transformations.  $\diamond$

In the same manner one can show:

**Proposition 7.** The mass term (38) with additional CP-symmetry breaking damping term is forminvariant under global isospin transformations.

However, invariance under local and global electroweak isospin transformations is a theoretical concept which is not met in physical reality. Although isospin symmetry breaking manifests itself on the phenomenological level only, the proper treatment of the phenomenological theory as an effective theory requires isospin symmetry breaking already on the parton level, i. e. in the parton propagator (1).

This symmetry violating parton propagator was extensively treated in [9], section 8.3. We adopt this symmetry breaking mass correction term from [9] and introduce it directly in the propagator equation. With inclusion of the CP-symmetry violating damping term and referred to the representation with charge-conjugated spinors, this equation reads

$$\begin{aligned} [i\gamma_{\alpha\alpha_1}^\mu \delta_{\kappa\kappa_1} \partial_\mu(x_1) - m_i^* \delta_{\alpha\alpha_1} \delta_{\kappa\kappa_1} - i\theta \delta_{\alpha\alpha_1} \gamma_{\kappa\kappa_1}^0 \\ + \theta' \delta_{\alpha\alpha_1} (\gamma^0 \gamma^3)_{\kappa\kappa_1}] \delta_{ii_1} F_{\alpha_1\alpha_2}^{\kappa_1\kappa_2}(x_1, x_2)_{i_1i_2} \\ = -iC_{\alpha\alpha_2} \gamma_{\kappa\kappa_2}^5 \lambda_i \delta_{ii_2} \delta(x_1 - x_2). \end{aligned} \quad (50)$$

Applying the method of the proofs of Propositions 5 and 6 one easily verifies:

**Proposition 8.** Equation (50) breaks CP- as well as global isospin invariance.



If the corresponding propagator is used for the derivation of an effective electroweak theory it causes also the violation of the local isospin invariance.

#### 4. Effective Dynamics of Physical Fields

For the treatment of the combined symmetry breaking we start from the canonical equations of motion for CP-symmetry breaking and discuss the effect of the additional global isospin symmetry breaking. In this proceeding symmetry breaking is exclusively expressed by the propagator, and in contrast to phenomenology no use is made of the Higg's formalism.

For CP-symmetry breaking the effective canonical equations of motion read (see [20], equations (59)–(62))

$$\partial_0 A_k^a = -E_k^a + \varepsilon_{kij} \partial_i G_j^a + \frac{1}{2} g_1 \eta^{abc} \varepsilon_{kij} (A_i^b G_j^c + G_i^b A_j^c), \quad (51)$$

$$\partial_0 G_k^a = B_k^a - \varepsilon_{kij} \partial_i A_j^a - \frac{1}{2} g_1 \eta^{abc} \varepsilon_{kij} (A_i^b A_j^c - G_i^b G_j^c), \quad (52)$$

$$\partial_0 E_k^a = \varepsilon_{kij} \partial_i B_j^a + g_1 \eta^{abc} \varepsilon_{kij} (A_i^b B_j^c - G_i^b E_j^c) - g_A^a J_k^a + \mu_A^2 A_k^a, \quad (53)$$

$$\partial_0 B_k^a = -\varepsilon_{kij} \partial_i E_j^a - g_1 \varepsilon_{kij} \eta^{abc} (G_i^b B_j^c + A_i^b E_j^c) - i g_G^a J_k^a - \mu_G^2 G_k^a, \quad (54)$$

where the currents are defined by  $J_a^\mu := \frac{1}{2} \bar{\psi} \sigma_a \gamma^\mu \psi$  and  $J_a^\mu := \frac{1}{2} \bar{\psi} \sigma_a (\gamma^5 \gamma^\mu) \psi$ ,  $a = 1, 2, 3, 0$ , with the coupling constants  $g_A^a = g_G^a = g$ ,  $a = 1, 2, 3$ , and  $g_A^0 = g_G^0 = g'$ , and where the various constants of the effective theory have been eliminated by the conditions [20], equation (63).

Provided the latter constants are evaluated in more detail, it is possible to attain (51)–(54) by renormalization with finite renormalization constants and without using the conditions [20], equation (63), i. e., the latter conditions are superfluous; see for instance such a renormalization procedure in [23], chapter 8.

The corresponding effective fermion equation follows from [20], equation (35) and reads

$$\begin{aligned} & [-i \gamma^\mu \partial_\mu + m] \psi_l \\ & + \frac{1}{2} [g \sigma_{ln}^a \gamma^k A_{ka} + g' \sigma_{ln}^0 \gamma^k A_{k0}] \psi_n \\ & + i \frac{1}{2} [g \sigma_{ln}^a (\gamma^k \gamma^5) G_{ka} + g' \sigma_{ln}^0 (\gamma^k \gamma^5) G_{k0}] \psi_n = 0. \end{aligned} \quad (55)$$

By an additional isospin symmetry breaking the mass term must be modified. In functional space the *full* term of the effective boson mass matrix is given by (cf. [20], equation (71))

$$\begin{aligned} m_{kl}^b b_k \partial_l^b + M_{kl}^b b_k \partial_l &\equiv 2 R_{II_1}^k m_{I_1 I_2}^f C_{I_2 I}^l b_k \partial_l^b \\ &- 6 W_{I_1 I_2 I_3 I_4} F_{I_4 K} R_{K I_1}^k C_{I_2 I_3}^l b_k \partial_l^b, \end{aligned} \quad (56)$$

where the first term on both sides stems from the spinorial mass matrix, while the second term contains the fermion propagator of the spinor field.

With regard to the calculation of the mass tensor (56) it is important to realize that in case of symmetry breaking the propagator as well as the boson states in (56) must be adapted to this situation. In particular for CP-violation the main effect on the boson states manifests itself in a change of the superspin-isospin group representations, whereas the mass corrections are very small. This result is drastically changed if a combined CP- and isospin symmetry breaking is considered: Then the combination of the modified boson superspin-isospin representation with the isospin symmetry breaking term of the propagator leads to massive changes in the mass tensor (56). One obtains

$$\begin{aligned} \mu_A^2 A_k^a &\rightarrow (\mu_A^2 \delta_{ab} - a_A \sigma_{ab}^2) A_k^b, \\ \mu_G^2 G_k^a &\rightarrow (\mu_G^2 \delta_{ab} + a_A \sigma_{ab}^2) G_k^b, \quad a = 1, 2, \end{aligned} \quad (57)$$

and

$$\begin{aligned} \mu_A^2 A_k^a &\rightarrow (\mu_A^2 \delta_{ab} + a_A \sigma_{ab}^1) A_k^b, \\ \mu_G^2 G_k^a &\rightarrow (\mu_G^2 \delta_{ab} - a_A \sigma_{ab}^1) G_k^b, \quad a = 3, 0. \end{aligned} \quad (58)$$

Due to the selfregularization of the spinor field all coefficients in (57) and (58) are finite without any further manipulation.

The nondiagonal form of the mass matrices shows that their vector fields are not identifiable with observable fields. The transition to observable fields can only be achieved by appropriate transformations. Compared with [20], equation (88), we use a slightly modified transformation by replacing  $y_{ab}$  by its complex conjugate  $y_{ab}^*$ . We suppress the star if this modified transformation is applied in the following.

Furthermore as the charge of the complex vector bosons depends on their status as ingoing or outgoing particles we use a new notation to avoid any association to the charge values of the W-bosons. From now on we define W and W<sup>+</sup>-bosons where the plus sign

means Hermitian conjugation (cf. for instance [14], equation (15.102), or [24], equation (12.43)). This gives the heuristic scheme

$$\begin{array}{cccc} \tilde{\mathbf{E}}_1 & \tilde{\mathbf{E}}_2 & \tilde{\mathbf{E}}_3 & \tilde{\mathbf{E}}_0 \\ \tilde{\mathbf{B}}_1 & \tilde{\mathbf{B}}_2 & \tilde{\mathbf{B}}_3 & \tilde{\mathbf{B}}_0 \\ \tilde{\mathbf{A}}_1 := \mathbf{W} & \tilde{\mathbf{A}}_2 := \mathbf{W}^+ & \tilde{\mathbf{A}}_3 := \mathbf{Z} & \tilde{\mathbf{A}}_0 := \mathbf{A} \\ \tilde{\mathbf{G}}_1 := \mathbf{M} & \tilde{\mathbf{G}}_2 := \mathbf{M}^+ & \tilde{\mathbf{G}}_3 := \mathbf{X} & \tilde{\mathbf{G}}_0 := \mathbf{G} \end{array} \quad (59)$$

where in contrast to the heuristic scheme in [20], equations (84) and (85), it is emphasized that for the electric as well as for the magnetic potentials only one set of electric and magnetic fields does exist.

These potentials and fields are assumed to diagonalize the mass matrices and are thus the physical quantities by definition. They are related to the original quantities by the transformations

$$\begin{aligned} E_k^a &= y_{ab} \tilde{E}_k^b, & B_k^a &= y_{ab} \tilde{B}_k^b, & A_k^a &= y_{ab} \tilde{A}_k^b, \\ G_k^a &= y_{ab} \tilde{G}_k^b, & a &= 1, 2, \end{aligned} \quad (60)$$

and

$$\begin{aligned} E_k^a &= z_{ab} \tilde{E}_k^b, & B_k^a &= z_{ab} \tilde{B}_k^b, & A_k^a &= z_{ab} \tilde{A}_k^b, \\ G_k^a &= z_{ab} \tilde{G}_k^b, & a &= 3, 0, \end{aligned} \quad (61)$$

where  $y_{ab}$  and  $z_{ab}$  are given by the unitary or orthogonal matrices, respectively,

$$y_{ab} = 2^{-1/2} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}, \quad z_{ab} = 2^{-1/2} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}. \quad (62)$$

The special form of  $z_{ab}$  in (62) results from  $g = -g' = 1$ , i. e., a Weinberg angle  $\Theta = 45^\circ$ . See the comments about the choice of this “idealized” angle in [20].

The transformations (60) and (61) can be considered as basis transformations of the isospin algebra which lead according to the representation of the fields in [20], equation (65) to the same transformations of all field quantities.

Furthermore apart from diagonalizing the mass matrices the unitary transformation  $y_{ab}$  is identical with that transformation in phenomenological theory which leads to the charged vector boson states. By means of these transformations one obtains the following canonical field equations from the set (51)–(54) for (i)  $a = 1, 2$ , or (ii)  $a = 3, 0$ , respectively:

$$\begin{aligned} \text{(i)} \quad \partial_0 \tilde{A}_k^a &= -\tilde{E}_k^a + \varepsilon_{kij} \partial_i \tilde{G}_j^a + (-1)^{a-1} g_1 \varepsilon_{kij} \\ &\quad \cdot 2^{-1/2} i [\tilde{A}_i^a (\tilde{G}_j^3 + \tilde{G}_j^0) + \tilde{G}_i^a (\tilde{A}_j^3 + \tilde{A}_j^0)], \\ \text{(ii)} \quad \partial_0 \tilde{A}_k^a &= -\tilde{E}_k^a + \varepsilon_{kij} \partial_i \tilde{G}_j^a \\ &\quad + g_1 \varepsilon_{kij} 2^{1/2} i [\tilde{A}_i^2 \tilde{G}_j^1 + \tilde{G}_i^2 \tilde{A}_j^1], \end{aligned} \quad (63)$$

$$\begin{aligned} \text{(i)} \quad \partial_0 \tilde{G}_k^a &= \tilde{B}_k^a - \varepsilon_{kij} \partial_i \tilde{A}_j^a - (-1)^{a-1} g_1 \varepsilon_{kij} \\ &\quad \cdot 2^{-1/2} i [\tilde{A}_i^a (\tilde{A}_j^3 + \tilde{A}_j^0) - \tilde{G}_i^a (\tilde{G}_j^3 + \tilde{G}_j^0)], \\ \text{(ii)} \quad \partial_0 \tilde{G}_k^a &= \tilde{B}_k^a - \varepsilon_{kij} \partial_i \tilde{A}_j^a \\ &\quad - g_1 \varepsilon_{kij} 2^{1/2} i [\tilde{A}_i^2 \tilde{A}_j^1 - \tilde{G}_i^2 \tilde{G}_j^1], \end{aligned} \quad (64)$$

$$\begin{aligned} \text{(i)} \quad \partial_0 \tilde{E}_k^a &= \varepsilon_{kij} \partial_i \tilde{B}_j^a - g_\chi \tilde{J}_k^a + m_A^a \delta_{ab} \tilde{A}_k^b \\ &\quad + (-1)^{a+1} g_1 \varepsilon_{kij} 2^{-1/2} i [(\tilde{B}_j^3 + \tilde{B}_j^0) \tilde{A}_i^a \\ &\quad - (\tilde{E}_j^3 + \tilde{E}_j^0) \tilde{G}_i^a - (\tilde{A}_i^3 + \tilde{A}_i^0) \tilde{B}_j^a \\ &\quad + (\tilde{G}_i^3 + \tilde{G}_i^0) \tilde{E}_j^a], \\ \text{(ii)} \quad \partial_0 \tilde{E}_k^a &= \varepsilon_{kij} \partial_i \tilde{B}_j^a - g_\chi \tilde{J}_k^a + m_A^a \delta_{ab} \tilde{A}_k^b \\ &\quad + g_1 \varepsilon_{kij} 2^{1/2} i [-\tilde{A}_i^1 \tilde{B}_j^2 + \tilde{A}_i^2 \tilde{B}_j^1 \\ &\quad + \tilde{G}_i^1 \tilde{E}_j^2 - \tilde{G}_i^2 \tilde{E}_j^1], \end{aligned} \quad (65)$$

$$\begin{aligned} \text{(i)} \quad \partial_0 \tilde{B}_k^a &= -\varepsilon_{kij} \partial_i \tilde{E}_j^a - i g_\pi \tilde{J}_k^a + m_G^a \delta_{ab} \tilde{G}_k^b \\ &\quad - g_1 (-1)^{a-1} \varepsilon_{kij} 2^{-1/2} i [(\tilde{B}_j^3 + \tilde{B}_j^0) \tilde{G}_i^a \\ &\quad + (\tilde{E}_j^3 + \tilde{E}_j^0) \tilde{A}_i^a - (\tilde{G}_i^3 + \tilde{G}_i^0) \tilde{B}_j^a \\ &\quad - (\tilde{A}_i^3 + \tilde{A}_i^0) \tilde{E}_j^a], \\ \text{(ii)} \quad \partial_0 \tilde{B}_k^a &= -\varepsilon_{kij} \partial_i \tilde{E}_j^a - i g_\pi \tilde{J}_k^a - m_G^a \delta_{ab} \tilde{G}_k^b \\ &\quad - g_1 \varepsilon_{kij} 2^{1/2} i [\tilde{G}_i^2 \tilde{B}_j^1 - \tilde{G}_i^1 \tilde{B}_j^2 \\ &\quad + \tilde{A}_i^2 \tilde{E}_j^1 - \tilde{A}_i^1 \tilde{E}_j^2]. \end{aligned} \quad (66)$$

The transformed currents are defined by  $\tilde{J}_k^a = y_{ab}^+ g_A^b j_k^b$  for  $a = 1, 2$  and  $\tilde{J}_k^a = z_{ab}^{-1} g_A^b j_k^b$  for  $a = 3, 0$ , with an analogous transformation for  $J_k^a$ . Furthermore in the transformed field equations (65) and (66) the transformed versions of the mass tensors (57) and (58) appear which are diagonal after the transformation. The corresponding diagonal elements  $m_A^a$  and  $m_G^a$ ,  $a = 1, 2, 3, 0$ , associated to the fields (59), are given by

$$m_A^a = \begin{pmatrix} \mu_A^2 - a \\ \mu_A^2 + a \\ \mu_A^2 - a \\ \mu_A^2 + a \end{pmatrix}, \quad m_G^a = \begin{pmatrix} \mu_G^2 + a \\ \mu_G^2 - a \\ \mu_G^2 + a \\ \mu_G^2 - a \end{pmatrix}. \quad (67)$$

For comparison with (67) we give the diagonal elements of the mass tensor for electric bosons if only isospin symmetry is broken (see [9], equations (8.74)–(8.82)):

$$m_A^a = \begin{pmatrix} \mu_A^2 \\ \mu_A^2 \\ \mu_A^2 + \mu K \\ \mu_A^2 - \mu K \end{pmatrix}. \quad (68)$$

In (68) in contrast to (67) the mass values of the charged vector bosons are invariant under charge conjugation. As in both cases isospin symmetry is broken the differences between the mass matrices (67) and (68) must result from CP-symmetry breaking. *But these differences are only indirectly generated by the relatively small CP-violating term in the propagator. Owing to this term the permutation invariance of the boson equation is broken which leads to parafermionic boson eigenstates being the real cause for this considerable change of the mass values.*

Finally we transform the fermion equation (55) to physical fields. In this connection attention has to be paid to the fact that isospin symmetry breaking acts also on the lepton masses which we express by the introduction of a diagonal lepton mass tensor  $\mathbf{m}$  in isospace. Then with (60)–(62) one gets

$$[-i\gamma^\mu\partial_\mu + \mathbf{m}]\psi + \frac{1}{2}g\hat{\sigma}^a[\gamma^k\tilde{A}_k^a + i(\gamma^k\gamma^5)\tilde{G}_k^a]\psi = 0, \quad (69)$$

where

$$\begin{aligned} \hat{\sigma}^a &:= y_{ab}^T g^b \sigma^b, & a = 1, 2, \\ \hat{\sigma}^a &:= z_{ab}^T g^b \sigma^b, & a = 3, 0 \end{aligned} \quad (70)$$

holds. These definitions differ from those of the transformed Pauli matrices in the field equations which are layed down by

$$\begin{aligned} \tilde{\sigma}^a &:= y_{ab}^+ g^b \sigma^b, & a = 1, 2, \\ \tilde{\sigma}^a &:= z_{ab}^T g^b \sigma^b, & a = 3, 0 \end{aligned} \quad (71)$$

with the corresponding currents  $\tilde{j}_\mu^a := \frac{1}{2}\bar{\psi}\tilde{\sigma}^a\gamma_\mu\psi$  and  $\tilde{J}_\mu^a := \frac{1}{2}\bar{\psi}\tilde{\sigma}^a(\gamma^5\gamma_\mu)\psi$ .

If electric as well as magnetic bosons occur an important question is whether for both species separate conservation laws can be derived. This is not the case. Independently of the above symmetry breaking only the conservation of the vector current, i. e. of the electric charge, can be established, and as the fermions are massive no conservation of the axial current is possible.

**Proposition 9.** For combined CP- and isospin symmetry breaking the effective canonical equations of motion (63)–(66) and the lepton Lagrangian (72) are forminvariant under the U(1)-group of charge transformations.

*Proof:* The U(1)-phase transformations of the whole system are fixed by the phase transformations of the charged currents, while the neutral currents remain invariant under such transformations. To verify this statement we consider the Lagrangian density, associated to the lepton equation (69),

$$\begin{aligned} \mathcal{L} &= \bar{\psi}[-i\gamma^\mu\partial_\mu + \mathbf{m}]\sigma^0\psi \\ &+ \frac{1}{2}g\hat{j}_k^a\tilde{A}_k^a + \frac{1}{2}g\hat{J}_k^a\tilde{G}_k^a \end{aligned} \quad (72)$$

with  $\hat{j}_k^a := \bar{\psi}\hat{\sigma}^a\gamma^k\psi$  and  $\hat{J}_k^a := \bar{\psi}\hat{\sigma}^a(\gamma^k\gamma^5)\psi$ . The corresponding Lagrangian density for (55) contains the currents  $j_k^a := \bar{\psi}\sigma^a\gamma^k\psi$  and  $J_k^a := \bar{\psi}\sigma^a(\gamma^k\gamma^5)\psi$ . According to [20], equation (41) the latter currents are hermitean. From this it follows that if one introduces the definitions  $j := \hat{j}^2$  and  $J := \hat{J}^2$  the hermitean conjugation of these quantities leads to  $j^+ = \hat{j}^1$  and  $J^+ = \hat{J}^1$ . Therefore one gets  $(\hat{j}^1\tilde{A}^1 + \hat{j}^2\tilde{A}^2) \equiv (j^+W + jW^+)$  and an analogous relation for the magnetic couplings.

Let us assume that the current  $j$  is transformed by a U(1)-phase transformation  $j' = Uj$ . Then  $j'^+ = U^{-1}j^+$  must hold. Furthermore the currents act as external forces in the field equations (63)–(66) which enforces the field quantities to transform in a definite way. In particular the  $W$ - and  $M$ -potentials and their associated fields are transformed with  $U$ , while their hermitean conjugates are transformed with  $U^{-1}$ . From this it follows that the Lagrangian density (72) is invariant under U(1)-phase transformations. By similar arguments the form invariance of (63)–(66) under these transformations can be verified.  $\diamond$

## 5. Preliminary Analysis of Experiments

The experiments of Urutskoev et al. are concerned with discharges between a solid metallic electrode and metallic foils embedded in a closed vessel filled with various fluids, in particular water. A great number of experiments and their numerical registrations have been reported [25]. To give a theoretical interpretation we concentrate on essential facts of the experimental outcome. According to Urutskoev [25] these experiments are characterized by the following facts:

“The electrodes are located in a blasting chamber, which is represented by a sealed thick metallic container the inner structure of which is made of high-density polyethylene. The electrodes consist of high-

purity titanium. The working fluid used was either bidistilled water with an impurity content of  $10^{-6}$  g/l or solutions of various metall salts in bidistilled water. In this experiment two banks capacitors with the total energy store  $W = 50$  kJ and the voltage  $U = 5$  kV are simultaneously discharged between the massive electrode and the two foil loads over a time of  $t = 0.1$  ms.

The main outcome is as follows. Analysis of the titanium foil remainder reveals a distorted isotopic ratio of titanium after the discharge. The natural titanium has the following composition:  $^{46}\text{Ti}$ , 8%;  $^{47}\text{Ti}$ , 7.3%;  $^{48}\text{Ti}$ , 73.8%;  $^{49}\text{Ti}$ , 5.5% and  $^{50}\text{Ti}$ , 5.4%. After the pulse part of Ti is lost. In particular, part of the stable  $^{48}\text{Ti}$  has not been transformed into another titanium isotope but has disappeared, while the  $^{46}\text{Ti}$ ,  $^{47}\text{Ti}$ ,  $^{49}\text{Ti}$  and  $^{50}\text{Ti}$  contents have remained in approximately the same ratio, of course with allowance made for experimental error. The deficiency of  $^{48}\text{Ti}$  in some experiments amounts to 10%, while the accuracy of measurements is for  $^{46,47,50}\text{Ti}$  0.2%, for  $^{48}\text{Ti}$  0.4% and for  $^{49}\text{Ti}$  0.13%. Simultaneously with the disappearance of  $^{48}\text{Ti}$  a sharp (10-fold) increase in the content of impurities in the sample was detected by mass spectrometry, X-ray fluorescence analysis and other methods. The percentage of the newly appeared impurities was proportional to that of the lost  $^{48}\text{Ti}$ .

Tables of the elements obtained in this way are given in [25].

In the experiments no neutron flux was observed neither there was any significant residual  $\gamma$ -activity registered. By the authors this is considered as an important evidence that strong interactions cannot be involved in the observed nuclear reactions. Thus one has to assume that only electroweak interactions must be responsible for the outcome of such experiments.

According to conventional experience  $^{48}\text{Ti}$  is an absolutely stable element with respect to electroweak decay. If in spite of this fact one considers the conditions for a hypothetical nuclear transmutation of  $^{48}\text{Ti}$  by electroweak processes, the next elements which are possible for a transition are  $^{48}_{21}\text{Sc}$  or  $^{48}_{23}\text{V}$ . Both these elements are considerably heavier than  $^{48}\text{Ti}$ , i. e., there is no energetic gradient to favour a decay. An estimate of the energy differences can be obtained by the application of the Weizsaecker mass formula [26], which holds also for the case where these elements are partly ionized.

In the nuclear shell model the 4s shell of  $^{48}\text{Ti}$  is occupied by two nucleons. If one assumes that by pair-

ing this shell contains two protons and that by electron capture one proton is transmuted into a neutron which forms a deuteron-like state with the remaining proton, then one gets for the energy difference between  $^{48}\text{Ti}$  as initial state and  $^{48}\text{Sc}$  as final state the value

$$(i) \ ^{48}_{22}\text{Ti} \rightarrow ^{48}_{21}\text{Sc} : E_i - E_f = -2.7 \text{ MeV},$$

while for  $^{48}\text{V}$  as final state the energetic difference reads

$$(ii) \ ^{48}_{22}\text{Ti} \rightarrow ^{48}_{23}\text{V} : E_i - E_f = -5.5 \text{ MeV}.$$

As due to the larger energy difference the process (ii) is more improbable than the process (i) we concentrate our discussion on the case (i). In conventional physics without symmetry breaking also the latter process has no chance of success, but we investigate this process under the conditions of combined CP- and isospin symmetry breaking.

For the process holds the reaction equation

$$e^- + u = \nu + d \text{ (electroncapture)}, \quad (73)$$

and for discharges this process can be considered as an irreversible transmutation of an electron into a neutrino which is accompanied by the simultaneous irreversible transmutation of an u-quark into a d-quark. Thus the whole process can be characterized by the time for the irreversible decay of the electron.

In order to obtain a correct result, our calculations will be performed with Gauss units and *not* with natural units.

To describe the process theoretically we start from the lepton equation (69). To include the hadron part we add the “electric” and the “magnetic” quark currents  $\tilde{h}_\mu^a$  and  $\tilde{H}_\mu^a$  to the lepton currents on the right-hand side of (65) and (66) where the quark currents result from a detailed calculation of the effective field equations.

Furthermore we consider the currents as the leading interaction terms in (65) and (66) and neglect in these equations the boson-boson interactions which are only corrections to the electron capture process. Then (65) and (66) can be exactly transformed into a representation where the vector potentials are expressed by their sources, i. e., the currents which are defined above.

In Gaussian units this leads to the following set of equations (cf. [20], equations (114), (115)):

$$c^{-2}\partial_t^2 \tilde{\mathbf{A}}^a + \nabla \times \nabla \times \tilde{\mathbf{A}}^a + \tilde{m}_a^2 \tilde{\mathbf{A}}^a = g_1(\tilde{\mathbf{j}}^a + \tilde{\mathbf{h}}^a) \quad (74)$$

and

$$c^{-2}\partial_t^2 \tilde{\mathbf{G}}^a + \nabla \times \nabla \times \tilde{\mathbf{G}}^a + \tilde{m}_a^2 \tilde{\mathbf{G}}^a = g_1(\tilde{\mathbf{J}}^a + \tilde{\mathbf{H}}^a) \quad (75)$$

with  $\tilde{m} := m\hbar^{-1}$ . If by means of Green functions these equations are resolved for the vector potentials, these solutions can be substituted in (69) providing in this way the starting point for the investigation of leptonic and semileptonic transitions.

But before starting any calculation some comments have to be made in order to clarify the physical meaning of this procedure.

(i) The effective canonical equations (51)–(55) stem from the Lagrangian in [20], equation (42). Although the SU(2)-gauge symmetry is broken, these equations are evaluated in “temporal gauge”, i.e. by putting  $A_0 = 0$ . By direct calculation it can be shown that this assumption is compatible with the Lagrangian formalism resulting from [20], equation (42) independently of any gauge invariance. In addition this assumption causes no loss of generality because (51)–(55) still admit the U(1)-gauge invariance of quantum electrodynamics where in [27] it was shown that with the temporal gauge the whole physics of quantum electrodynamics is covered. Besides in conventional theory one starts with an SU(2)-gauge invariance where the temporal gauge is a physical gauge. The Higgs coupling must be gauge-invariant, because renormalization depends on this property. Therefore in this gauge the whole theory is formulated without  $A_0$  and the Higgs coupling can only lead to the creation of non-trivial longitudinal boson states which means that no loss of generality can be expected from this side too, as the longitudinal branch belongs to the variables of the temporal gauge.

(ii) Substitution of the solutions of (74) and (75) into (69) shows that the irreversible dynamics of lepton states is produced by various leptonic and semileptonic couplings. This means that the simultaneous possibility of transitions into different channels leads to a competition between the corresponding processes. In particular the weak processes compete with optical transitions of the electron. As long as these optical transitions dominate the decay times no weak transitions can be observed. The interplay of the various irreversible transitions has been thoroughly discussed and theoretically formulated in [28], chapter 1. An application of this formalism would go beyond the scope of this paper. So we treat only the electron capture in detail and compare the resulting electron decay times with known optical transition times.

(iii) The physics of the vector bosons depends on the modulus of their mass values as well as on the phase of these values. For free vector bosons real mass values

mean that they are ordinary massive particles, whereas for imaginary values they must be tachyons.

Within the Lagrange formalism the following relations can be established: If in the Lagrange function ([20], equation (42)) the masses  $\mu_A$  and  $\mu_G$  are real, then in the canonical equations for the electric and magnetic field strengths real values for  $\mu_A$  and  $\mu_G$  appear. This leads to real masses for the wave equations of free electric vector bosons, but in the wave equations of free magnetic vector bosons to imaginary masses, cf. [20], equation (58). As the latter equations characterize the physical properties of the vector bosons the corresponding mass values are the physical relevant ones, whereas  $\mu_A$  and  $\mu_G$  in [20], equation (42) are parameters. In order to formulate a Lagrangian density which describes electric as well as magnetic vector bosons as massive particles  $\mu_A$  must be real, while  $\mu_G$  must be imaginary in [20], equation (42). This leads to real masses in the canonical equations for the electric field strengths and to imaginary masses in the equations for the magnetic field strengths, whereas both types of wave equations exhibit real mass values.

To decide which case has been realized, we return to the canonical equations (51)–(54) which are a direct consequence of the weak mapping procedure. In (53) the mass is real, while in (54) the mass is imaginary. This means according to our scheme that electric as well as magnetic electroweak vector bosons must be massive particles (apart from the photon). This statement has, however, to be considered with caution:

The “mass” values in (53) and (54) stem from an exact algebraic evaluation of the weak mapping formulas, whereas the results of the corresponding orbital integrals in the mapping formulas, cf. [12], equations (44), (49), are formfactors which have been idealized by (unknown) constants. Only for electric bosons such integrals have been studied in more detail by several authors. These studies show that  $\mu_A$  must be real. For magnetic bosons comparable studies are lacking. So the question cannot be answered with confidence whether the magnetic bosons are massive particles or tachyons.

For the process under consideration the energetically most favourable exchange boson is the magnetic M-boson the mass of which is minimized in parallelism with the minimizing of the electric photon mass in the scheme (67). Owing to (75) we assume that this boson will act as a massive particle, although for this mass value a final proof has to be delivered by a more detailed calculation.

## 6. Decay Rates of Electron Capture

According to (ii) of the foregoing discussion we consider only the isolated decay process of electron capture instead of treating the complete irreversible dynamics of leptonic and semileptonic processes. To derive the corresponding decay time we apply a method developed by Heitler [22]. The latter method is based on the Schroedinger picture, i. e., in this case on the use of the generalized Dirac equation (69).

For a first estimate of this process we use a simplified version of (75) for  $a = 1$ . This equation reads

$$(c^{-2}\partial_t^2 - \Delta + \tilde{m}_1^2)\tilde{G}_k^1(\mathbf{r}, t) = g_1\bar{q}(\mathbf{r}, t)(\gamma_k\gamma^5)\tilde{\sigma}_1 q(\mathbf{r}, t), \quad (76)$$

where  $q(\mathbf{r}, t) := q_{\alpha A}(\mathbf{r}, t)$  are the quark states and  $A$  is the isospin index. For eigenstates of energy one obtains

$$\bar{q}(\mathbf{r}, t)(\gamma_k\gamma^5)\tilde{\sigma}_1 q(\mathbf{r}, t) = \bar{q}_A(\mathbf{r})(\gamma_k\gamma^5)(\tilde{\sigma}_1)_{AB} q_B(\mathbf{r}) \exp(i\omega_{AB}t) \quad (77)$$

with  $\omega_{AB} = \hbar^{-1}(E_A - E_B)$ . Then by the ansatz  $\mathbf{G} = \mathbf{G}(\mathbf{r}) \exp(i\omega_{AB}t)$ , (76) goes over into

$$(-\omega_{AB}^2 c^{-2} + \tilde{m}_1^2 - \Delta)\tilde{G}_k^1(\mathbf{r}) = 4\pi g_1 \bar{q}_A(\mathbf{r}) \gamma_k \gamma^5 (\tilde{\sigma}_1)_{AB} q_B(\mathbf{r}). \quad (78)$$

The latter equation can be solved by a time-independent Green function. So the complete time-dependent solution reads

$$\tilde{G}_k^1(\mathbf{r}, t) = 4\pi i g_1 \int d^3r' G(\mathbf{r} - \mathbf{r}') q_A^\dagger(\mathbf{r}') \cdot (\gamma^0 \gamma_k \gamma^5) (\tilde{\sigma}_1)_{AB} q_B(\mathbf{r}') \exp[i\omega_{AB}t]. \quad (79)$$

If (79) is substituted in (69) and if in this equation all coupling terms for competing processes are left out, then one obtains

$$\begin{aligned} i\hbar\partial_t\psi(\mathbf{r}, t) &= [i\hbar c(\gamma^0\gamma^k)\partial_k + \gamma^0 m_e c^2]\psi(\mathbf{r}, t) \\ &+ 4\pi g_1^2 (\gamma^0\gamma^k\gamma^5)\tilde{\sigma}_1\psi(\mathbf{r}, t) \int d^3r' G(\mathbf{r} - \mathbf{r}') \\ &\cdot \bar{q}_A(\mathbf{r}')(\gamma_k\gamma^5)(\tilde{\sigma}_1)_{AB} q_B(\mathbf{r}') \exp[i\omega_{AB}t] \\ &= [H_0 + H_1]\psi(\mathbf{r}, t). \end{aligned} \quad (80)$$

With  $H_0\psi_n = E_n\psi_n$  and  $\psi = \sum_n b_n(t)\psi_n(\mathbf{r}) \exp(i\omega_n t)$  one gets from (80)

$$\begin{aligned} -i\hbar\partial_t b_n(t) &= \sum_m (H_1)_{nm} b_m(t) \exp[i(\omega_m - \omega_n)t], \end{aligned} \quad (81)$$

which for isospinors  $\psi_1 = \psi_{nu}$ ,  $\psi_2 = \psi_e$  and  $q_1 = q_u$ ,  $q_2 = q_d$  with  $\tilde{\sigma}_1 = (\sigma_1 + i\sigma_2)$  and  $\tilde{\sigma}_1 = (\sigma_1 - i\sigma_2)$  yields

$$(H_1)_{nm} = 4\pi g_1^2 \int d^3r d^3r' \psi_\nu^\dagger(\mathbf{r})_f (\gamma^0 \gamma^k \gamma^5) \psi_e(\mathbf{r})_i \cdot G(\mathbf{r} - \mathbf{r}') q_d^\dagger(\mathbf{r}')_f (\gamma^0 \gamma_k \gamma^5) q_u(\mathbf{r}')_i \exp[i\omega_{du}t], \quad (82)$$

where  $i$  means ingoing and  $f$  outgoing final states. As the process proceeds at low energies, the spin dependence of lepton and quark states can be approximated by the use of Pauli spinors. This leads to the definition of the spin matrix elements

$$\mathcal{M}(s_1, s_2, s_3, s_4) := \sum_k u^+(s_1)(\gamma^0 \gamma^k \gamma^5) u(s_2) \cdot u^+(s_3)(\gamma^0 \gamma_k \gamma^5) u(s_4), \quad (83)$$

where  $u(s_i)$ ,  $i = 1, 2, 3, 4$ , are the Pauli spinors for leptons and quarks. Then (82) can be expressed in the form

$$(H_1)_{nm} := h_{\nu d, eu}^1 \exp[i\omega_{du}t] \quad (84)$$

with  $n = \nu d$  and  $m = eu$  and

$$\begin{aligned} h_{\nu d, eu}^1 &= 4\pi g_1^2 \int d^3r d^3r' \varphi_\nu^*(\mathbf{r}) \varphi_e(\mathbf{r}) \\ &\cdot G(\mathbf{r} - \mathbf{r}') \chi_d^*(\mathbf{r}') \chi_u(\mathbf{r}') \mathcal{M}. \end{aligned} \quad (85)$$

For the special case of electron capture the general set of equations (81) goes over into the set

$$\begin{aligned} -i\hbar\dot{b}_{eu} &= \sum h_{eu, \nu d}^1 b_{\nu d} \exp[i(\omega_{\nu e} + \omega_{du})t], \\ -i\hbar\dot{b}_{\nu d} &= h_{\nu d, eu}^1 b_{eu} \exp[i(\omega_{e\nu} + \omega_{ud})t], \end{aligned} \quad (86)$$

which one can try to solve by the ansatz

$$b_{eu}(0) = 1, \quad b_{\nu d}(0) = 0, \quad b_{eu}(t) = e^{-\gamma t}. \quad (87)$$

This gives for the decay constant  $\gamma$  the equation

$$\begin{aligned} i\gamma &= \sum_\nu h_{eu, \nu d}^1 h_{\nu d, eu}^{1+} [1 - \exp[i(\omega_{\nu e} + \omega_{du} - i\gamma)t]] \\ &\cdot \hbar^{-2} [\omega_{e\nu} + \omega_{ud} + i\gamma]^{-1}. \end{aligned} \quad (88)$$

Owing to the lepton and quark wave functions in the matrix elements (85) and the summation over the final

neutrino states, the formula (88) contains a lot of “hidden” information. For a numerical evaluation this content of information cannot be incorporated in one step in the calculation. Rather several steps are necessary.

(i) In the first step we eliminate the quark states from (85). The electrodes of the capacitor have a finite extension and the charges are located at the surfaces of the opposite electrodes. While the electron states must be subjected to the resulting boundary conditions, the quark states are confined to the interior of the nuclei and are practically independent of these conditions. Thus we consider a representative of the nuclei which is located at the origin  $\mathbf{r}' = 0$ . Furthermore we assume that in the transition  $u \rightarrow d$  only the isospin parts of the quark states will be changed, whereas the orbital parts remain unchanged. This means that in (85) the normalized orbital density of the quark states appears which owing to the concentration in the nucleus can be approximated by a delta-distribution.

If in (85) the Fourier transform of the Green function is introduced this yields

$$h_{\nu d,eu}^1 = (4\pi)^2 (2\pi)^{-3} g_1^2 \int d^3 r d^3 k \varphi_\nu^*(\mathbf{r}) \varphi_e(\mathbf{r}) \cdot (\mathbf{k}^2 + \mu^2)^{-1} \exp(-i\mathbf{k}\mathbf{r}) \mathcal{M} \quad (89)$$

with  $\mu^2 = \tilde{m}_a^2 - \omega_{ud}^2 c^{-2}$ .

(ii) In the next step we treat the electron part. We expand the electron state in (89) into a Fourier series, where the plane waves are defined by

$$\varphi(\mathbf{r}, \mathbf{p}) := V^{-1/2} \exp[i(\mathbf{p}\mathbf{r})(\hbar c)^{-1}], \quad (90)$$

and where the wave vectors  $\mathbf{p}$  are chosen to have the dimension of energy which makes subsequent calculations easier. The normalization volume  $V$  will be specified later.

In the interval  $[\mathbf{p}, \mathbf{p} + d\mathbf{p}]$  the number of plane waves in the volume  $V$  is given by

$$dN = dp_1 dp_2 dp_3 V (2\pi\hbar c)^{-3}. \quad (91)$$

Hence the Fourier decomposition of  $\psi_e(\mathbf{r})$  reads

$$\psi_e(\mathbf{r}) = \int d^3 p V^{1/2} (2\pi\hbar c)^{-3} \cdot \tilde{\psi}_e(\mathbf{p}) \exp[i(\mathbf{p}\mathbf{r})(\hbar c)^{-1}]. \quad (92)$$

As in (88) the sum over the neutrino states is extended over a complete set of states one can choose any suitable set which in our case are the plane waves (90)

themselves. Substituting (92) and the plane neutrino waves into the matrix element (89) one gets

$$h_{\nu d,eu}^1 = \int d^3 p_e (2\pi\hbar c)^{-3} \tilde{\psi}_e(\mathbf{p}_e) \cdot g_1^2 [(\mathbf{p}_e - \mathbf{p}_\nu)^2 (\hbar c)^{-2} + \mu^2]^{-1} \mathcal{M}. \quad (93)$$

If in accordance with (88) the square of the modulus of  $h^1$  is formed, it contains the statistical operator  $\tilde{\psi}_e(\mathbf{p}_e) \tilde{\psi}_e^*(\mathbf{p}'_e)$  which is nondiagonal.

But during the discharge a great number of processes proceeds which differ only in the phase of their initial electron states. Thus for getting a mean decay time a phase average of the statistical operator can be performed leading to its diagonalization. Defining

$$f(\mathbf{p}_e) = |\tilde{\psi}_e(\mathbf{p}_e)|^2, \quad (94)$$

the formula (88) reads after phase averaging

$$i\hbar\bar{\gamma} = \int d^3 p_e (2\pi\hbar c)^{-3} f(\mathbf{p}_e) \lambda(\mathbf{p}_e) \quad (95)$$

with

$$\begin{aligned} \lambda = & \sum_{\nu} (4\pi)^4 g_1^4 [(\mathbf{p}_e - \mathbf{p}_\nu)^2 (\hbar c)^{-2} + \mu^2]^{-2} \\ & \cdot \mathcal{M} \mathcal{M}^* V^{-2} [1 - \exp[i(\omega_{\nu e} + \omega_{du} - i\gamma)t]] \\ & \cdot \hbar^{-1} [\omega_{e\nu} + \omega_{ud} + i\gamma]^{-1}. \end{aligned} \quad (96)$$

In addition to phase averaging (96) can be averaged over all spin states of ingoing and outgoing particles. By direct calculation one obtains for the matrix elements (83) the relation

$$\sum_{s_1 s_2 s_3 s_4} \mathcal{M}(s_1 s_2 s_3 s_4) \mathcal{M}(s_1 s_2 s_3 s_4)^+ = 12, \quad (97)$$

which for this summation leads to the intermediate decay constant  $\bar{\lambda}$ .

(iii) In this step we sum (integrate) over the momenta of the outgoing neutrino states. The number of the neutrino plane waves in the volume  $V$  in the intervall  $[p_\nu, p_\nu + dp_\nu]$  is given by the general formula (91). Neglecting the small neutrino mass one gets  $E_\nu = p_\nu \equiv |\mathbf{p}|_\nu$  and thus

$$dN_\nu = E_\nu^2 dE_\nu d\Omega V (2\pi\hbar c)^{-3}. \quad (98)$$

If one replaces the symbolic sum over  $\nu$  by the integral over  $E_\nu$  and  $\Omega$  and expresses the  $\omega$  terms by the

corresponding energies, then the spin-averaged equation (92) can be rewritten in the form

$$\begin{aligned} \bar{\lambda} = & \int dE_\nu d\Omega (2\pi)^{-3} (\hbar c) \\ & \cdot [(\mathbf{p}_e^2 - 2|\mathbf{p}_e|E_\nu \cos \Theta + E_\nu^2) + \mu^2(\hbar c)^2]^{-2} \\ & \cdot (4\pi)^4 12V^{-1} g_1^4 E_\nu^2 \\ & \cdot [1 - \exp[i(E_\nu + E_d - E_e - E_u)t/\hbar] e^{\gamma t}] \\ & \cdot (E_e + E_u - E_\nu - E_d + i\gamma\hbar)^{-1}. \end{aligned} \quad (99)$$

To calculate the integral over  $E_\nu$  we apply Heitler's integral formula ([22], equation (9)). In the case of (99) this yields

$$\begin{aligned} \bar{\lambda} = & i\pi \int d\Omega 192(2\pi) g_1^4 V^{-1} (\hbar c) E_0^2 \\ & \cdot [(\mathbf{p}_e^2 - 2|\mathbf{p}_e|E_0 \cos \Theta + E_0^2) + \mu^2(\hbar c)^2]^{-2} \end{aligned} \quad (100)$$

with  $E_0 = E_e + E_u - E_d$  and the condition  $E_0 > 0$ .

In (100) the remaining integral over  $\Omega$  can be carried out exactly. If by means of the relations  $E_e^2 = \mathbf{p}_e^2 + m_e^2 c^4$  and  $\mu^2(\hbar c)^2 = [m_a^2 c^4 - (E_u - E_d)^2]$  the resulting formula is rearranged one obtains the expression

$$\begin{aligned} \bar{\lambda} = & i96(2\pi)^3 g_1^4 (\hbar c) E_0^2 \{[(m_a^2 - m_e^2)c^4 \\ & + 2E_e E_0]^2 - (2E_{\text{kin}} E_0)^2\}^{-1} \end{aligned} \quad (101)$$

with  $E_{\text{kin}} = (E_e^2 - m_e^2 c^4)^{1/2}$ .

Substitution of (101) into (95) yields

$$\begin{aligned} \bar{\gamma} = & \int d^3p (2\pi\hbar c)^{-3} f(p) 96g_1^4 c [E_e + \Delta E]^2 \\ & \cdot \{[(m_a^2 - m_e^2)c^4 + 2E_e(E_e + \Delta E)]^2 \\ & - [2E_{\text{kin}}(E_e + \Delta E)]^2\}^{-1} \end{aligned} \quad (102)$$

with  $E_0 = E_e + \Delta E$  and  $\Delta E = E_u - E_d$ . Formula (102) does not contain the volume  $V$ . Hence one can choose an arbitrary large volume without changing  $\bar{\gamma}$ . In contrast to this arbitrariness of the general volume, the electrons themselves are in located states which must satisfy the corresponding boundary conditions.

(iv) During the discharge the electrons which are emitted by the cathode hit on the anode. The latter consists of a metal lattice with a surface layer of positively charged ions. It is this layer where the electrons are captured before reacting by further processes.

We assume a standard volume of this surface layer of  $1 \text{ cm}^2 \cdot 10^{-8} \text{ cm}$ . Without knowing details of the discharge process, we assume that the electrons are uniformly distributed in this layer, i. e.,  $\psi_e(\mathbf{r}) = v^{-1/2}$  within the layer with  $v = a_1 \times a_2 \times a_3$ . Then one gets for the Fourier transform of  $\psi_e$

$$\tilde{\psi}_e(\mathbf{p}) = \prod_{i=1}^3 a_i^{-1/2} \sin(k_i a_i) (k_i a_i)^{-1} \quad (103)$$

with  $k_i = p_i(\hbar c)^{-1}$ . For  $f(\mathbf{p})$  defined in (94) this yields

$$f(\mathbf{p}) = \prod_{i=1}^3 a_i^{-1} [\sin(k_i a_i)]^2 (k_i a_i)^{-2}. \quad (104)$$

For convenience we choose  $a_1 = a_2 = 1 \text{ cm}$  and  $a_3 = 10^{-8} \text{ cm}$  which leads to the final formula

$$\begin{aligned} \bar{\gamma} = & \int dk_1 dk_2 dk_3 a^{-1} [\sin(k_1)]^2 [\sin(k_2)]^2 \\ & \cdot [\sin(k_3 a_3)]^2 (k_1)^{-2} (k_2)^{-2} (k_3)^{-2} \\ & \cdot 96g_1^4 c [E_e + \Delta E]^2 \{[(m_a^2 - m_e^2)c^4]^2 \\ & + 2E_e(E_e + \Delta E) - [2E_{\text{kin}}(E_e + \Delta E)]^2\}^{-1}. \end{aligned} \quad (105)$$

In spite of our simplifications (105) is a relative complicated expression which in the general case can be evaluated only with computer assistance. To illustrate the meaning of this formula we consider a special case which exceptionally can be exactly integrated.

(v) For  $m_a = m_e$  we use the relation  $E_e^2 - E_{\text{kin}}^2 = m_e^2 c^4$  and obtain from (105)

$$\begin{aligned} \bar{\gamma} = & \int dk_1 dk_2 dk_3 768a_3^{-1} [\sin(k_1)]^2 (k_1)^{-2} \\ & \cdot [\sin(k_2)]^2 (k_2)^{-2} [\sin(k_3 a_3)]^2 \\ & \cdot (k_3 a_3)^{-2} g_1^4 c [m_e^2 c^4]^{-1}. \end{aligned} \quad (106)$$

The outgoing neutrino must have a positive energy. This is only possible if the ingoing electron has an energy  $E_e$  which fulfills the condition  $E_e + \Delta E \geq 0$ . For the transition of a u-quark into a d-quark, i. e. the transition of a proton into a neutron  $\Delta E$  is negativ. For instance we choose  $\Delta E = -1.5 \text{ MeV}$ . Then it is  $E_e \geq 1.5 \text{ MeV}$  and the lower limit of the  $k$ -integral results from  $k^2(\hbar c)^2 = E_e^2 - m_e^2 c^4$ .



In this case one gets  $k = 0.75 \cdot 10^{11} \text{ cm}^{-1}$ . To simplify matters we put  $k_1 = k_2 = 0$ ,  $k_3 = k$  as lower limit of the integration in (106). The corresponding integrals can be exactly solved which yields for (106)

$$\bar{\gamma} = 128\pi^2 10^{13} g_1^4 c (m_e^2 c^4)^{-1}. \quad (107)$$

If the coupling constant  $g_1$  is identified with the phenomenological constant  $g_{CC}$ , then with  $g_1^2 \equiv g_{CC}^2 = e^2(1.73)^{-1}$  (see [29], p. 71) the formula (106) yields  $\bar{\gamma} \approx 10^1$  for one nucleus. In the layer are  $10^{16}$  nuclei or ions, respectively. Hence for an electronic

wave function which extends over the whole layer one obtains

$$\bar{\gamma} = 10^{17} \text{ s}^{-1}, \quad \bar{\tau} = 10^{-17} \text{ s}. \quad (108)$$

In this case the competing optical transitions are suppressed in favour of the electroweak decay.

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